

# Power System Economic Dispatch Using Traditional and Neural Networks Programs

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**Abstract**— The introduction of techniques of artificial intelligence in software of control and decision is an essential element in research and development of tomorrow's power systems. Neural networks are among the techniques most used in the field of artificial intelligence. The economic dispatch is a key sector in the electricity networks, where it must generate less energy for the same demand with good economic operation reducing repartition grid losses to have the least cost of kWh possible. In this paper, we will opt for a quicker economic dispatch; we will program a mesh network of 8 buses including 3 generation units using traditional program then backpropagation learning neural network program, finally, we will compare the two programs in terms of speed and reliability.

**Index Terms**— Power systems, economic dispatch, artificial intelligence, neural networks, grid losses, traditional program, backpropagation learning.

## 1 ECONOMIC DISPATCH

### 1.1 Introduction

THE Economic dispatch is a static optimization problem which consists in distributing active power production requested by different grid buses from generation unites in the most economical way. This distribution must of course respect the limits of production of generation units. The variable to be optimized is the production cost.

### 1.2 The cost function

The cost of production of a plant is generally modeled by a polynomial function of second degree in  $P_{Gi}$  (active power generated by the plant  $i$ ) whose coefficients are constants specific to each plant: [1]

$$C_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (1)$$

### 1.3 Economic dispatch solution

To minimise the total production cost of an interconnected power system we must minimize the sum of cost functions of production units

$$\text{Minimise} \quad C = \sum_{i=1}^{ng} C_i(P_{Gi}) \quad (2)$$

taking into consideration the following constraints:

$$\text{Equality constraints:} \quad \sum_{i=1}^{ng} P_{Gi} = \sum_{j=1}^{nd} P_{Dj} \quad (3)$$

$$\text{Inequality constraints:} \quad P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (4)$$

where  $C$  is total cost function,  $ng$  is a total number of producer nodes and  $nd$  is a total number of consumer nodes.  $P_{Gi}$  represents the active power generated by the  $i$ th generator,  $P_{Dj}$  is the active power consumed by the  $j$ th load,  $P_{Gi}^{\max}$  is the maximum active power of the  $i$ th generator and  $P_{Gi}^{\min}$  is the mini-

mum active power of the  $i$ th generator. [2]

The solution of this problem is obtained by using the Lagrange function which is obtained by multiplying the function of equality constraints by the Lagrange multiplier  $\lambda$ , adding to the total cost function (5).

$$L(P_{Gi}, \lambda) = C + \lambda \left( \sum_{i=1}^{ng} P_{Gi} - \sum_{j=1}^{nd} P_{Dj} \right) \quad (5)$$

The derivatives of the Lagrange equation with respect to each independent variable ( $P_{Gi}, \lambda$ ) give us: [3]

$$\frac{\delta L}{\delta P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda = 0 \Rightarrow \lambda = \frac{dC_i(P_{Gi})}{dP_{Gi}} \quad (6)$$

$$\frac{\delta L}{\delta \lambda} = \sum_{j=1}^{nd} P_{Dj} - \sum_{i=1}^{ng} P_{Gi} = 0 \quad (7)$$

So, from (6)  $\lambda$  represents the incremental cost of the  $i$ th generator, then, for each energy packet the generator having the least  $\lambda$  is responsible of production (least cost) respecting the constraints of (3) and (4).

### 1.4 Insertion of losses formula in the economic dispatch

#### 1.4.1 Calculation of Losses ( $P_L$ ):

The general formula of losses following the equations of power flows is:

$$P_L = \psi^T G \psi \quad (8)$$

with  $\psi = M \delta$ .  $M$  and  $\delta$  are matrices of *line's incidence* and *phases of nodes* respectively.  $G$  is the diagonal matrix of line conductances.

$$G = \text{diag} [G_{12} G_{13} G_{14} \dots G_{(n-1)n}] \quad (9)$$

$\delta$  can be approximated by a DC Load Flow so,

$$P_G - P_D = A \delta \Rightarrow \delta = A^{-1} (P_G - P_D) \quad (10)$$

$A$  represents the DC Load flow Matrix, therefore,

$$P_L = P_D^T B P_D - 2P_D^T B P_G + P_G^T B P_G \quad (11)$$

where

$$B = A^{-1} M^T G M A^{-1}$$

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### 1.4.2 Penalty Factor: supposing

$P_{Gi}$  Power generated by the  $i$ th plant.

$P_{Ci}$  Part of the power generated that is really consumed by loads.

$P_{Li}$  Part of the power generated that is lost in the lines, we know that:

$$P_{Gi} = P_{Ci} + P_{Li} \quad (12)$$

and from(1) 
$$\frac{dC}{dP_{Ci}} = b_i + 2c_i P_{Gi} \quad (13)$$

by substitutions 
$$\frac{dC}{dP_{Ci}} = b'_i + 2c'_i P_{Gi} \quad (14)$$

where  $b'_i = b_i f_i$  &  $c'_i = c_i f_i$  are the new coefficients of (1), with  $f_i = (1 - \frac{dP_{Li}}{dP_{Ci}})^{-1}$  is the penalty factor of the incremental cost.

### 1.4.3 Criterion of convergence:

If  $\left| \sum_{i=1}^{ng} P_{Gi} - \sum_{j=1}^{nd} P_{Dj} - P_L \right| \leq \epsilon$  the system has converged. [1]

## 2 NEURAL NETWORKS

### 2.1 Introduction

The origin of artificial neural networks comes from the biological neuron modeling test by Warren McCulluch and Walter Pitts. They assumed that the nerve impulse is the result of a simple calculation made by each neuron and the thought is born with the collective effect of a network of interconnected neurons. [4]

### 2.2 Neuron Model

A neuron consists essentially of an integrator that performs the weighted sum of its inputs. The result  $n$  of this sum is then transformed by a transfer function  $f$  which produces the neuron output  $a$ . Fig. 1

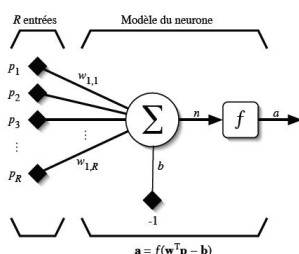


Fig. 1. Artificial neuron model

$$n = \sum_{j=1}^R w_{1,j} p_j - b \quad (15)$$

$$= w_{1,1} p_1 + w_{1,2} p_2 + \dots + w_{1,R} p_R - b$$

This output corresponds to a weighted sum of the weights  $w_{ij}$  and inputs  $p_i$  minus the bias  $b$ . [5]

### 2.3 Neural network learning

There are essentially two types of learning, unsupervised and supervised. In our paper we will use a supervised learning

where we impose to the network specific operations by forcing it from inputs submitted, the outputs to take by changing the synaptic weights. [6]

### 2.4 Error backpropagation learning

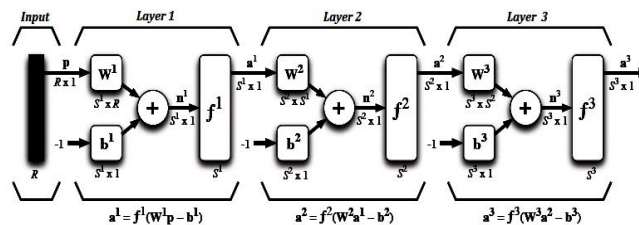


Fig. 2 Representation of three layers network

The simple perceptron consists of a single layer of  $S$  neurons which are fully connected to vector  $p$  of  $R$  entries. In a multilayer perceptron the equation that describes the output of layer  $k$  (Fig. 2) is given by:

$$a^k = f^k (W^k a^{k-1} - b^k) \text{ for } k = 1, \dots, M \quad (16)$$

where  $M$  is the total number of layers. The network outputs correspond to  $a^M$ . The backpropagation algorithm uses the mean squared error as performance index, and allows a supervised learning with a set of associations (stimulus, target)  $\{(p_q, d_q)\}$ ,  $q=1, \dots, Q$  where  $p_q$  represents the stimulus vector (inputs) and  $d_q$  the target vector (desired outputs). At each time  $t$ , we can forwardpropagate a different stimulus  $p(t)$  through the network of Fig. 2 to obtain an output vector  $a(t)$ . This allows us to calculate the error  $e(t)$  between what the network produces as output for the stimulus and the target  $d(t)$  associated with it:

$$e(t) = d(t) - a(t) \quad (17)$$

The performance index  $F$  minimizes the mean square error. This index is approximated by the instantaneous error  $\hat{F}(x)$ , the vector  $x$  includes all the weights and biases of the network,

$$\hat{F}(x) = e^T(t)e(t) \quad (18)$$

we use the *gradient descent method* to optimize  $x$ :

$$\Delta w_{i,j}^k(t) = -\eta \frac{\partial \hat{F}}{\partial w_{i,j}^k}; \quad \Delta b_i^k(t) = -\eta \frac{\partial \hat{F}}{\partial b_i^k} \quad (19)$$

where  $\eta$  represents the learning rate, so:

$$\frac{\partial \hat{F}}{\partial w_{i,j}^k} = \frac{\partial \hat{F}}{\partial n_i^k} \times \frac{\partial n_i^k}{\partial w_{i,j}^k} \quad (20)$$

$$\frac{\partial \hat{F}}{\partial b_i^k} = \frac{\partial \hat{F}}{\partial n_i^k} \times \frac{\partial n_i^k}{\partial b_i^k} \quad (21)$$

$n_i^k$  represent the *activation levels* of a layer  $k$  which depend directly on weights and bias on this layer;

$$n_i^k = \sum_{j=1}^{s^{k-1}} w_{i,j}^k a_j^{k-1} - b_i^k \quad (22)$$

so, the second term of (20) and (21) becomes:

$$\frac{\partial n_i^k}{\partial w_{i,j}^k} = a_j^{k-1}; \quad \frac{\partial n_i^k}{\partial b_i^k} = -1 \quad (23)$$

for the first term of (20) and (21), we define the  $ty_{s_i^k}$  where  $s_i^k = \frac{\partial \hat{F}}{\partial n_i^k}$ , then (19) becomes:

$$\Delta w_{i,j}^k(t) = -\eta s_i^k(t) a_j^{k-1}(t); \Delta b_i^k(t) = \eta s_i^k(t) \quad (24)$$

with

$$s^k = \left( \frac{\partial n^{k+1}}{\partial n^k} \right)^T \frac{\partial \hat{F}}{\partial n^{k+1}} = \hat{F}^k(n^k) (W^{k+1})(s^{k+1}) \quad (25)$$

In this case, we get a recursive formula where sensitivity layers upstream (input  $s^k$ ) depend on the sensitivity of the layers downstream (output  $s^{k+1}$ ). That's where the term «back-propagation» comes, because the direction of information propagation is reversed compared to that of (16). [5]

### 3 ECONOMIC DISPATCH USING NEURAL NETWORKS

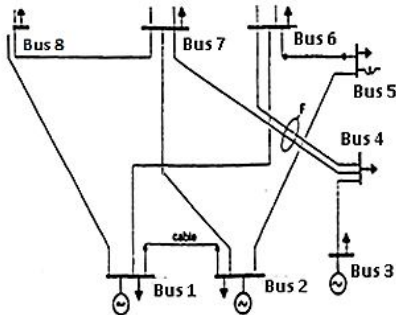


Fig. 3 The 138KV area from IEEE 24-Bus Test Network[7]

In this example we will study and program the economic dispatch of a mesh network using the traditional and the neural network programs. The network is the 138KV area in the IEEE 24-Bus Test Network (Fig. 3) and constituted of 8 busbars; three Generation buses (Bus 1, Bus 2 and Bus 3) and five Load buses (Bus 4, Bus 5, Bus 6, Bus 7 and Bus 8) [7]

First, we show the characteristics of generators;

TABLE 1  
CHARACTERISTICS OF GENERATORS [7]

| Gen Bus | Pmax [MW] | Pmin [MW] | a [\$ /h] | b [\$ /MWh] | c [\$ /MW²h] |
|---------|-----------|-----------|-----------|-------------|--------------|
| Bus 1   | 308       | 0         | 646.99    | 19.18       | 0.0322       |
| Bus 2   | 350       | 0         | 646.99    | 19.18       | 0.0322       |
| Bus 3   | 250       | 0         | 1829.71   | 27.22       | 0.0628       |

Second, we give busbars distances and impedances;

TABLE 2  
DISTANCES AND IMPEDANCES BETWEEN BUSBARS [7]

| Line | from | to | Distance [Miles] | R[Ω]  | X [Ω] |
|------|------|----|------------------|-------|-------|
| 1    | 1    | 8  | 55               | 0.055 | 0.21  |
| 2    | 1    | 6  | 45               | 0.02  | 0.08  |
| 3    | 1    | 2  | 3                | 0.003 | 0.014 |
| 4    | 2    | 7  | 60               | 0.015 | 0.115 |
| 5    | 2    | 5  | 50               | 0.05  | 0.192 |
| 6    | 3    | 4  | 16               | 0.016 | 0.06  |
| 7    | 4    | 7  | 43               | 0.043 | 0.165 |

|    |   |   |    |       |       |
|----|---|---|----|-------|-------|
| 8  | 4 | 6 | 43 | 0.043 | 0.165 |
| 9  | 5 | 6 | 16 | 0.014 | 0.061 |
| 10 | 7 | 8 | 31 | 0.031 | 0.119 |

Afterwards, we present energy demands of load buses;

TABLE 3  
ENERGY DEMANDS OF LOAD BUSES

| DEMAND [MW] |     |     |     |     | TOTAL DEMAND [MW] |
|-------------|-----|-----|-----|-----|-------------------|
| B4          | B5  | B6  | B7  | B8  |                   |
| 0           | 46  | 1   | 15  | 90  | 152               |
| 0           | 56  | 10  | 25  | 100 | 191               |
| 0           | 66  | 20  | 35  | 110 | 231               |
| 0           | 76  | 30  | 45  | 120 | 271               |
| 0           | 86  | 40  | 55  | 130 | 311               |
| 0           | 96  | 50  | 65  | 140 | 351               |
| 0           | 106 | 60  | 75  | 150 | 391               |
| 10          | 116 | 70  | 85  | 160 | 441               |
| 20          | 126 | 80  | 95  | 170 | 491               |
| 30          | 136 | 90  | 105 | 180 | 541               |
| 40          | 146 | 100 | 115 | 190 | 591               |
| 50          | 156 | 110 | 125 | 200 | 641               |
| 60          | 166 | 120 | 135 | 210 | 691               |
| 70          | 176 | 130 | 145 | 220 | 741               |
| 80          | 186 | 140 | 155 | 230 | 791               |
| 90          | 196 | 150 | 165 | 240 | 841               |
| 100         | 206 | 160 | 175 | 250 | 891               |

#### 3.1 Traditional Program

The traditional program (dispatch with losses) gives us the results shown in Table 4.

#### 3.2 Associated Neural Network Program

Based on the results of the traditional dispatch program, we will create a neural network with P: input matrix [5x17] taken from Table 3, T: target matrix [3x17] taken from Table 4 representing the power generated by the three stations using traditional program. We will use two layers neural network with error backpropagation learning (Fig. 4):

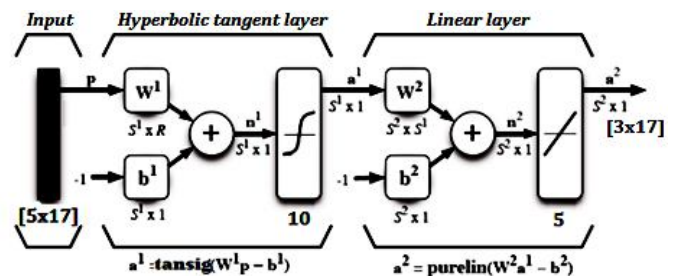


Fig. 4 Representation of the neural network used

The network contains: an input (P matrix), a hidden layer of 10 neurons with tansig (hyperbolic tangent sigmoid) activation

function and an output layer of 5 neurons with Pureline (linear) activation function. The learning results are shown in Table 4.

TABLE 4

RESULTS OF PLANTS GENERATIONS FOLLOWING TOTAL DEMANDS OF TABLE 3 USING TRADITIONAL AND NEURAL NETWORK PROGRAMS

| TRADITIONAL PROGRAM RESULTS [MW] (target) |         |         | NEURAL NETWORK PROGRAM RESULTS [MW] (output) |         |         |
|---|---------|---------|--|---------|---------|
| B1  | B2      | B3      | B1   | B2      | B3      |
| 153.053                                   | 0       | 0       | 152.451                                      | 1.859   | 1.622   |
| 192.352                                   | 0       | 0       | 192.758                                      | 1.464   | 0.589   |
| 232.705                                   | 0       | 0       | 235.009                                      | 0.292   | 0.631   |
| 273.112                                   | 0       | 0       | 272.580                                      | 0.599   | 1.231   |
| 307.207                                   | 6.291   | 0       | 300.584                                      | 8.783   | 1.924   |
| 307.633                                   | 46.175  | 0       | 310.004                                      | 38.134  | 1.909   |
| 288.103                                   | 106.935 | 0       | 288.254                                      | 107.243 | 0.371   |
| 307.412                                   | 117.122 | 21.097  | 298.709                                      | 124.038 | 21.392  |
| 307.91                                    | 127.326 | 61.005  | 307.791                                      | 126.953 | 60.894  |
| 307.444                                   | 137.548 | 102.136 | 309.066                                      | 131.349 | 105.869 |
| 307.271                                   | 150.482 | 140.448 | 310.001                                      | 150.728 | 137.788 |
| 307.745                                   | 180.552 | 160.551 | 309.189                                      | 180.089 | 160.627 |
| 307.244                                   | 211.715 | 180.666 | 307.778                                      | 210.695 | 181.743 |
| 307.769                                   | 241.992 | 200.792 | 307.894                                      | 239.753 | 203.061 |
| 307.322                                   | 273.376 | 220.930 | 309.280                                      | 269.228 | 222.915 |
| 307.900                                   | 303.865 | 241.08  | 310.083                                      | 303.132 | 238.014 |
| 307.906                                   | 349.977 | 244.867 | 308.067                                      | 346.016 | 244.817 |

TABLE 5

BUSBAR DEMANDS FOR NEURAL NETWORK TESTING

| Ptest [MW] |     |     |     |     |
|------------|-----|-----|-----|-----|
| B4         | B5  | B6  | B7  | B8  |
| 0          | 75  | 29  | 44  | 119 |
| 0          | 77  | 31  | 46  | 121 |
| 11         | 117 | 71  | 86  | 161 |
| 25         | 131 | 85  | 100 | 175 |
| 53         | 159 | 113 | 128 | 203 |
| 65         | 171 | 125 | 140 | 215 |
| 72         | 178 | 132 | 147 | 222 |
| 86         | 192 | 146 | 161 | 236 |

then,  $P_{test}$  is passed through the traditional and Neural network programs. This will give us Ttest and Ytest respectively (Table 6).

TABLE 6

TEST RESULTS MATRICES TTEST AND YTEST

| Ttest [MW] |         |         | Ytest [MW] |         |         |
|------------|---------|---------|------------|---------|---------|
| B1         | B2      | B3      | B1         | B2      | B3      |
| 269.069    | 0       | 0       | 269.784    | 0.001   | 0.004   |
| 277.156    | 0       | 0       | 276.264    | 0.971   | 0.003   |
| 307.460    | 118.142 | 25.079  | 309.767    | 116.283 | 24.360  |
| 307.172    | 132.435 | 82.044  | 307.092    | 128.125 | 86.656  |
| 307.892    | 189.592 | 166.584 | 307.629    | 189.400 | 167.042 |
| 307.503    | 226.841 | 190.727 | 307.038    | 228.499 | 189.488 |
| 307.878    | 248.060 | 204.819 | 306.984    | 250.808 | 202.875 |
| 307.666    | 291.657 | 233.018 | 307.694    | 290.522 | 234.198 |

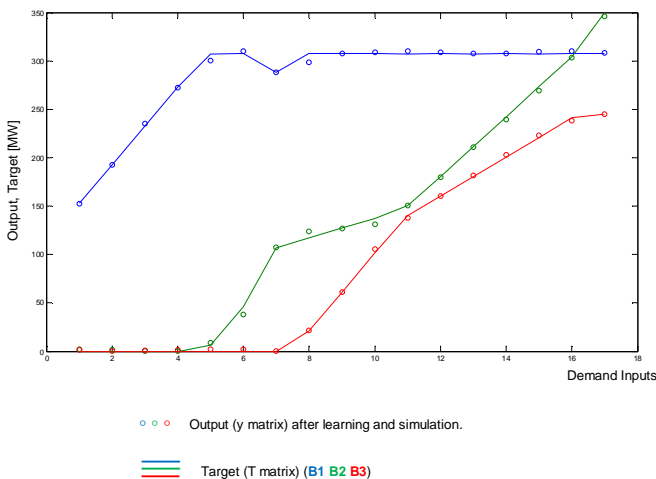


Fig. 5 Comparison between output (Y) and Target (T)

We can see from Table 4 and Fig. 5 that the values of the output are close to the target. We can deduce then that the training of our neural network is considered good. We must therefore test the network to judge the reliability of the learning.

### 3.3 Network Test

We introduce the matrix  $P_{test}$  which contains values in the borders of training matrix P, but did not participate in it.

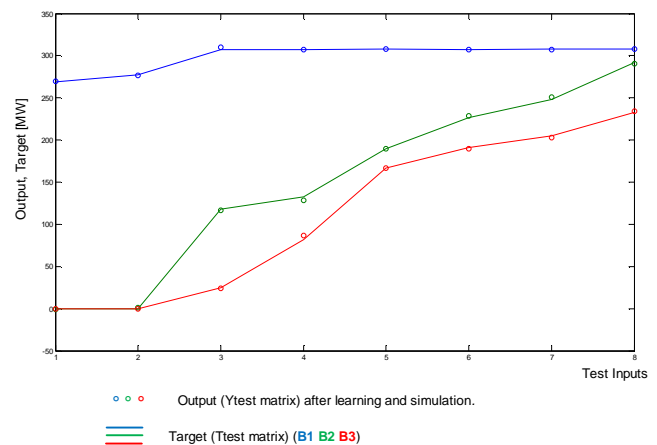


Fig. 6 Comparison between Ytest and Ttest

Based on data in Table 6 and Fig. 6, we can confirm that the learning of our neural network is considered good. To have a very good to excellent learning, output must be perfectly identical to the target ( $Y \equiv T$ ), this requires:

- Using a good economic dispatch traditional program, based on precise data: power plants parameters, lines imped-

ances and grid topography;

- Simulating the largest number of cases based on actual statistics of busbars energy demands;
- Simulating random and unpredictable cases (fault cases) which are not included in the statistics;
- Reducing learning step of the neural network to make a significant interpolation for unknown data (unpredictable);
- Adding more data, i.e. increasing the size of P and T matrices, thus making a more detailed learning which covers the majority of cases from the traditional program (caution: very lengthy P and T matrices ask for more powerful processors otherwise we risk having slow program).

A good economic dispatch software based on neural networks, must have a complete and excellent learning and test, then, it could execute directly the dispatching, i.e. demand data could be presented in real time to the neural network program where the decisions take two cases: if the neural network finds data (real time) in its matrix P, it gives immediate release to predefined outputs from matrix T; if the neural network cannot find data in matrix P, it makes a direct interpolation to give results.

#### 4 CONCLUSION

The most important difference between traditional and neural network programs is the execution time. The traditional program is slow because:

- It uses iterative loops : *if*, *while* and *for* ; which affects directly the execution time;
- It stocks a largest number of parameters: parameters of plants, lines and each busbar;
- The more a network is meshed the more the program is slow.

For the neural network program, the execution time is very rapid (milliseconds), because:

- It contains loops only on training; then after training the problem of dispatching becomes a classification problem.
- It is an executor; it performs data already stored and interpolates intermediate ones.

In practice, we opt for an economic dispatch with the fastest possible frequency (5 or 15 minutes instead of every hour [8]), even in real time [9], which is practically impossible with the traditional program.

To control power grids, time is very important and vital especially in economic dispatch and many other disciplines: protection, stability, power flow, etc. Using techniques of artificial intelligence and more specifically neural networks reduces the execution time; this will bring a huge economic gain by reducing losses, thus restraining the consumption of fuels (coal, oil, gas, uranium etc.). The decrease of production implies a contribution to the preservation of the environment by reducing pollution and global warming.

#### 5 ACKNOWLEDGMENT

The authors gratefully acknowledge the contributions of Peter Kelen, research officer and owner of Power Optimization Software.

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